The Cost-Effective Choice of Policy Instruments to Cap Aggregate Emissions with Costly Enforcement*

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Abstract

We study the cost-effectiveness of inducing compliance in a program that caps aggregate emissions of a given pollutant from a set of heterogeneous firms based on emissions standards and the relative cost-effectiveness of such a program with respect to an optimally designed program based on tradable discharge permits. Our analysis considers abatement, monitoring and sanctioning costs, as well as perfect and imperfect information on the part of the regulator with regard to the polluters' abatement costs. Under perfect information we find that (a) the total cost-effective design of a program based on standards is one in which the standards are firm specific and perfectly enforced, and (b) the

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total cost of an optimally designed program based on standards is lower than the total cost of an optimally designed transferable emission permits system, except under special conditions. This is true when it is optimum to induce perfect compliance and when it is not. Under imperfect information, nevertheless, it is only with a system of tradable permits that is perfectly enforced with a constant marginal penalty tied to the price of the permits that the regulator can surmount the informational problem and at the same time minimize the total cost of the program with certainty.

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1 Introduction

One of the most important features behind any emissions control policy is the total cost of the implied aggregate abatement. Environmental economists have been giving a clear policy recommendation for this issue for a long time: whenever possible, a regulator should cap emissions by means of a competitive market on emission permits because this policy minimizes the aggregate abatement costs of reaching any chosen cap. This policy recommendation has had its impact. The European Union adopted an Emissions Trading Scheme (EU - ETS) as an important instrument to limit its emissions of greenhouse gases. Until the appearance of the EU - ETS, the US was home to the major emissions trading program, the federal SO_2 allowance market to control acid rain, and other regional markets such as those for NOx and SOx under the RECLAIM program in southern California. Other regulatory programs based on transferable emission permits have been implemented in other regions as well. One example is Santiago de Chile's Emissions Compensation Program, a market for emission capacity of total suspended particles.

The apparent success of this policy recommendation may be surprising, though, because abatement costs are not the only social costs of capping emissions. Other relevant costs include the enforcement costs.[1]. Interestingly, the literature has not yet given a definite answer as to the relative cost-effectiveness of a tradable emission permits system with respect to one based on emission standards when enforcement costs are brought into the picture.[2] Malik (1992) compares the costs of reaching a given level of aggregate emissions by means of a perfectly enforced program based on uniform emission standards with that of a perfectly enforced program based on tradable permits, for a regulator with perfect information. He concludes that the enforcement costs under tradable permits may be higher than those under emission

[1] We use the term enforcement to refer to the set of actions to induce compliance. These actions include, among others, monitoring emissions and sanctioning detected violations.

[2] A recent paper surveying the literature on the choice of policy instruments completely omits this issue (see Goulder and Parry, 2008).

standards. Therefore, although the program based on tradable permits minimizes the aggregate abatement costs, the total costs of such a program could end up being higher than the total costs of a program based on emission standards. Malik does not consider sanctioning costs because he focuses on perfectly enforced programs. Hahn and Axtell (1995) compare the relative costs of a uniform emission standard with that of a tradable permits system allowing non-compliance, but considering only abatement costs and fines. These authors do not consider monitoring or sanctioning costs. More recently, Chávez, et al. (2009) extend Malik's contribution for a regulator that, unlike Malik's, cannot perfectly observe the abatement costs of the firms, but instead knows their distribution. With this information, the regulator chooses to inspect all firms with a homogeneous probability that is high enough to assure the compliance of the firms with higher abatement costs. The authors prove that emissions standards are more costly than tradable permits with this monitoring strategy.

One important aspect that most of the existing work share is that they do not consider the cost-effectiveness of inducing compliance. They merely assume that perfect compliance is the regulator's objective, as in Malik (1992) and Chávez et al. (2009), or that it is simply non-attainable, as in Hahn and Axtell (1995). Stranlund (2007) seems to be the first to have addressed the issue of whether the regulator can use non-compliance as a way to reduce the costs of a program that caps aggregate emissions. To put it clearly, the question he addresses is the following: if a regulator wants to achieve a certain level of aggregate emissions from a set of firms at the least possible cost using tradable permits, must the regulator design a program that allows a certain level of non-compliance or must the program be enforced perfectly? The answer depends on the relative marginal cost of inspecting versus sanctioning, which, in turn, depends on the structure of the penalty function. Taking into account abatement, monitoring, and sanctioning costs, Stranlund concludes that the regulator could always decrease the costs of a program that allows non-compliance with an increasing marginal penalty inducing full compliance with a constant marginal penalty.[3] Arguedas (2008) addresses the same question for the case of an emission standard, a regulator with perfect information, and only one firm. She concludes that "if the regulator is entitled to choose the structure of the fine, linear penalties are socially preferred and the optimal policy induces compliance" (p. 155). The analysis of one firm fails, nevertheless, to illustrate a central aspect of the design of cost-effective regulation in the real world; namely, how does the regulator must allocate emissions responsibilities and monitoring and sanctioning efforts among different firms in order to minimize the total cost of the pollution control program.

In this paper, we first extend Arguedas' (2008) analysis to derive the condition under which it is cost-effective to induce compliance in a system of emissions standards with more than one regulated firm, possibly firm-specific monitoring and sanctioning costs, and perfect information. Considering the total costs of the program (abatement, monitoring, and sanctioning), we then characterize the total cost-effective design of an emission standards system and compare it to the costs of an optimally designed transferable emissions permit system, as in Stranlund (2007), under different assumptions of the penalty structure.

Doing this, we find that the cost-effective design of a program that caps aggregate emissions of a given pollutant from a set of firms based on emissions standards is one in which standards are firm-specific and perfectly enforced. In addition, we find that an optimally designed system of tradable permits minimizes the total costs of attaining a certain level of aggregate emissions only under special circumstances. This is basically because the distribution of emissions generated by the market for permits and its corresponding cost-effective monitoring differ from the distribution of emissions and monitoring efforts that minimizes the total costs of the program.

[3] In a recent work, Stranlund et al. (2009) analyze the optimality of perfect compliance for the case of emission taxes.

This result holds both in the case when it is cost-effective to induce compliance and when it is cost-effective to allow violations.

We then extend our analysis to derive the condition under which it is expected-cost-effective to induce perfect compliance in a system of emission standards and imperfect information. We find that it is precisely under imperfect information that the relative advantage of tradable permits arises. In effect, our final results suggest that it is only under tradable permits and a constant marginal penalty that the regulator can surmount the informational problem and implement the total cost-minimizing design of the emissions control program.

The paper is organized as follows. In section 2, we present the standard model of compliance behaviour of a risk-neutral polluter firm that faces an emission standard. We use this model to derive the condition under which it is cost-effective for a regulator to induce perfect compliance in a system of emissions standards that caps the aggregate emissions under perfect information. Section 3 contains a characterization of the cost-effective design of such a program, both when it is cost-effective to induce perfect compliance and when it is not. We then characterize the cost-minimizing design of a program based on emissions standards when the regulator chooses the structure of the penalty function. In Section 4, we compare the costs of a program based on standards with that of a program based on tradable permits. Section 5 deals with the case of imperfect information. Finally, we present our conclusions in Section 6.

2 The Cost-Effectiveness of Inducing Perfect Compliance

In this section, we answer the following question: when is it cost-effective for a regulator to induce perfect compliance? In order to respond, we first present the standard model of compliance behaviour of a risk-neutral polluter firm under an emission standard (See Malik 1992; Harford 1978). From this model, we derive the emissions level with which the firm responds to the regulation. We then present the problem that a total cost-minimizing regulator solves, taking into account the firms' best responses, when designing a program that caps aggregate emissions by setting standards. From this model, we derive the condition under which it is cost-effective for the regulator to allow perfect compliance.

2.1 Compliance behaviour of a firm under an emission standard

Assume that reducing emissions of a given pollutant e is costly for a firm. The (minimum) abatement cost function for this firm, which we will call firm i, is $c_i(e_i)$, where e_i is its level of emissions.[4] The abatement cost function is assumed to be strictly decreasing and convex in the firm's emissions e [$c_i'(e_i) < 0$ and $c_i''(e_i) > 0$].

The firm faces an emission standard (a maximum allowable level of emissions) s_i. An emissions violation v_i occurs when the firm's emissions exceed the emissions standard: $v_i = e_i - s_i > 0$; otherwise the firm is compliant. The firm is audited with probability π_i. An audit provides the regulator with perfect information about the firm's compliance status. If the firm is audited and found in violation, a penalty $f(v_i)$ is imposed. Following Stranlund (2007), throughout the paper, we assume that the structure of the penalty function is $f(e_i - s_i) = \phi(e_i - s_i) + \frac{\gamma}{2}(e_i - s_i)^2$, with $\phi > 0$ and $\gamma \geq 0$.

Under an emissions standard, a firm i chooses the level of emissions to minimize the total expected compliance cost, which consists of the firm's abatement costs plus the expected penalty. Thus, firm $i's$ problem is to choose the level of emissions to

[4] Firms' abatement costs can vary for many reasons, including differences in the type of goods produced, the techniques and technologies of production and emissions control, input and output prices, and other more specific issues related to the corresponding industrial sector. Note that some of these factors may not be completely observable for a regulator.

solve

$$\min_{e_i} c_i(e_i) + \pi_i f\left(e_i - s_i\right) \tag{1}$$
$$\text{subject to } e_i - s_i \geq 0$$

The Lagrange equation for this problem is given by $\Gamma_i = c_i(e_i) + \pi_i f\left(e_i - s_i\right) - \eta_i\left(e_i - s_i\right)$, with η_i being the Lagrange multiplier. The set of necessary Kuhn-Tucker conditions for a positive level of emissions is:

$$\frac{\partial \Gamma_i}{\partial e_i} = c_i'(e_i) + \pi_i f'\left(e_i - s_i\right) - \eta_i = 0 \tag{2a}$$
$$\frac{\partial \Gamma_i}{\partial \eta_i} = -e_i + s_i \leq 0; \eta_i \geq 0; \eta_i\left(e_i - s_i\right) = 0 \tag{2b}$$

The above conditions show that the firm is going to comply with the standard if the expected marginal penalty is not lower than the marginal abatement cost associated with an emissions level equal to the emissions standard. That is, $e_i = s_i$ if $-c_i'(s_i) \leq \pi_i f'(0)$. Otherwise, the firm is going to choose a level of emissions $e_i(s_i, \pi_i) > s_i$, where $e_i(s_i, \pi_i)$ is the solution to $-c_i'(e_i) = \pi_i f'\left(e_i - s_i\right)$.

2.2 The Condition under which it is Cost-Effective for a Regulator to induce Perfect Compliance

Now assume a regulator who is in charge of implementing a pollution control program based on emissions standards. The objective of the program is to cap the aggregate level of emissions of a given pollutant to a level T. The regulator wants to achieve this target at the least cost. Ths costs of the program are comprised of the abatement costs of the firms and the regulator's monitoring and sanctioning costs. Thus, for every firm i the regulator selects the probability of inspection π_i and the emission standard s_i. There are n firms that emit this pollutant. Following the literature we first assume that the regulator has perfect information regarding the abatement costs of the firms. (We remove the assumption of perfect information in Section 5). Based on this information, the regulator solves the following problem:

$$\min_{\substack{(s_1,s_2,..,s_n)\\(\pi_1,\pi_2,..,\pi_n)}} \sum_{i=1}^{n} \left(c_i(e_i) + \mu_i\pi_i + \beta_i\pi_i f(e_i - s_i)\right) \tag{3a}$$

subject to:

$$e_i = e_i(s_i, \pi_i) \tag{3b}$$

$$\sum_{i=1}^{n} e_i(s_i, \pi_i) = T \tag{3c}$$

$$s_i \leq e_i \ \forall i = 1, ...n \tag{3d}$$

The regulator minimizes the total expected cost of the program. This is comprised of the aggregate abatement costs $\sum_{i=1}^{n} c_i(e_i)$, the aggregate monitoring costs, $\sum_{i=1}^{n} \mu_i\pi_i$, with μ_i being the cost of inspecting firm i, and the expected aggregate sanctioning costs, $\sum_{i=1}^{n} \beta_i\pi_i f(e_i - s_i)$, assuming that sanctioning firm i has a cost of β_i per dollar of fine. For the moment, we assume that the structure of the penalty function $f(e_i - s_i)$ is out of the control of the environmental regulator. The first constraint incorporates the fact that the regulator knows that firm i will react to a standard s_i and a monitoring probability π_i according to its best response function $e(s_i, \pi_i)$. The second constraint summarizes the environmental objective of the program, namely, that the aggregate level of emissions must be equal to a predetermined target T. Finally, the third constraint acknowledges that it may be in the interest of the firms to violate the emission standard. The Lagrange of the regulator's problem can be written as

$$L = \sum_{i=1}^{n} \left(c_i(e_i) + \mu_i\pi_i + \beta_i\pi_i f(e_i - s_i)\right) + \lambda_1 \left[\sum_{i=1}^{n} e_i - E\right] + \sum_{i=1}^{n} \lambda_2^i (s_i - e_i)$$

with λ_1 and λ_2^i being the $n+1$ multipliers. The $n \times 2 + n + 1$ necessary Kuhn-Tucker conditions for positive levels of the standard and the auditing probability are:

$$\frac{\partial L}{\partial s_i} = c_i'(e_i)\frac{\partial e_i}{\partial s_i} + \beta_i \pi_i f'(e - s_i)(\frac{\partial e_i}{\partial s_i} - 1) + \lambda_1 \frac{\partial e_i}{\partial s_i} + \lambda_2^i (1 - \frac{\partial e_i}{\partial s_i}) = 0, i = 1, ..., n \tag{4}$$

$$\frac{\partial L}{\partial \pi_i} = c_i'(e_i)\frac{\partial e_i}{\partial \pi_i} + \mu_i + \beta_i \left(f(e_i - s_i) + \pi_i f'(e - s_i)\frac{\partial e_i}{\partial \pi_i} \right) + \lambda_1 \frac{\partial e_i}{\partial \pi_i} - \lambda_2^i \frac{\partial e_i}{\partial \pi_i} = 0, i = 1, ..., n \tag{5}$$

$$\frac{\partial L}{\partial \lambda_1} = \sum_{i=1}^{n} e_i - E = 0 \tag{6}$$

$$\frac{\partial L}{\partial \lambda_2^i} = s_i - e_i \leq 0, \lambda_2^i \geq 0, \lambda_2^i \times (s_i - e_i) = 0 \tag{7}$$

We assume that these conditions are necessary and sufficient to characterize the optimal solution of the problem. Using these conditions, we derive the following Proposition:

Proposition 1 *When the penalty structure is given, the cost-effective design of a pollution control program that caps aggregate emissions using emissions standards calls the regulator to induce all firms to comply with the standards if and only if*

$$\mu_i \frac{f''(0)}{f'(0)} \leq \beta_i f'(0) \tag{8}$$

for all i. If this condition is not met and the regulator wants to achieve the cap cost-effectively, it should allow those plants for which $\mu_i \frac{f''(0)}{f'(0)} > \beta_i f'(0)$ to violate the emission standards.

Proof of Proposition 1: see Appendix.

The right-hand side of (8) is the marginal increase in the sanctioning costs when the regulator marginally decreases the standard. The left-hand side is the marginal decrease in monitoring costs that the regulator can attain by decreasing the monitoring probability accordingly so as to leave the level of emissions unchanged. Therefore, what the condition is saying is the following: it is not cost-effective for the regulator to move the standard and the monitoring probability so as to make the firm to marginally violate the standard when it is complying with it, if this increases the sanctioning costs more than it decreases the monitoring costs. When this is the case, the regulator should set π_i and s_i so as to induce the firm to comply with the standard. Otherwise, allowing the firm to violate the standard will increase the costs of the program.

Our Proposition 1 is an extension of Arguedas' (2008) Proposition 1 to the case of n firms and heterogeneous monitoring and sanctioning costs ($\mu_i \neq \mu_j$ and $\beta_i \neq \beta_j$, for at least some $i \neq j$, $i, j = 1, ..n$). It is also analogous to the condition derived by Stranlund (2007) for the case of transferable permits, but with homogeneous monitoring and sanctioning costs. Therefore, we can conclude that the condition under which it is cost-effective for a regulator to induce compliance is not instrument-dependent. Proposition 1 also says that when monitoring and sanctioning costs differ among firms, it could be cost-effective for the regulator to allow some firms to violate the standards while inducing compliance in the rest. This result cannot be observed when monitoring and sanctioning costs are the same for all firms. In this case, the regulator must induce all firms to comply or to violate. There are several reasons why the regulator's monitoring and sanctioning costs may differ among firms. Stranlund et al. (2009) mention the distance between the firm and the enforcing agency, the variation in the production technologies within and between industry sectors, and the number of discharge points per plant as possible reasons why the regulator's monitoring costs may differ between firms. The latter could be an example of a firm's investment to conceal non-compliance (Heyes 2000). At the same time, the imposition of penalties can motivate firms to engage in costly activities to contest enforcement actions (Jost 1997a, 1997b). Consequently, sanctioning costs may differ between firms because of their differing propensity to litigate sanctions and challenge the legislation (Kambhu 1989).

3 The cost-minimizing design of a program based on emission standards

We now turn to characterize the expected cost-minimizing design of a program that controls pollution with emission standards. We do this for the case in which the penalty structure is out of the control of the environmental regulator and for the case in which it is not.

3.1 A given penalty function

When the penalty structure is exogenously given to the regulator, condition (8) dictates whether or not it is cost-effective to induce perfect compliance. In the first case, we know from Chávez et al. (2009) and Malik (1992) that the optimal policy $(\pi_1^*, \pi_2^*, ...\pi_n^*, s_1^*, s_2^*, ...s_n^*)$ that induces perfect compliance is characterized by:

$$c_i'(s_i^*) + \mu_i \frac{d\pi_i^*}{ds_i} = c_j'(s_j^*) + \mu_j \frac{d\pi_j^*}{ds_j}, \text{ for all } i \neq j, (i,j) = 1, ..., n, \tag{9}$$

$$\text{and } \pi_i^* = \frac{-c_i'(s_i^*)}{f'(0)}, \text{for all } i = 1, ..., n.$$

Equations (9) tell that when it is cost-effective to induce compliance, the regulator has to set emission standards such that the *sum* of marginal abatement and monitoring costs are equal between firms.

When (8) does not hold, a regulator interested in minimizing the social costs of a program that caps aggregate emissions to a certain level has to design such program (meaning to choose the auditing probability and the emission standard for each firm) so as to allow a certain level of non-compliance.[5] In other words, the cost-minimizing standards must be set such that $e_i > s_i^*$ for all i. From the Kuhn-Tucker

[5] The analysis of this paper is static. Therefore, it does not consider the potential dynamic effects of allowing non-compliance, which were suggested to us by a Reviewer of this journal. We acknowledge that, if the regulator tolerates a certain level of violation, this could trigger a

condition (7), this implies that $\lambda_2^i = 0$. It follows that the relevant Kuhn-Tucker conditions in this case are

$$\frac{\partial L}{\partial s_i} = c_i'(e_i)\frac{\partial e_i}{\partial s_i} + \beta_i \pi_i f'(e_i - s_i)(\frac{\partial e_i}{\partial s_i} - 1) + \lambda_1 \frac{\partial e_i}{\partial s_i} = 0$$

$$\frac{\partial L}{\partial \pi_i} = c_i'(e_i)\frac{\partial e_i}{\partial \pi_i} + \mu_i + \beta_i \left[f(e_i - s_i) + \pi_i f'(e - s_i)\frac{\partial e_i}{\partial \pi_i} \right] + \lambda_1 \frac{\partial e_i}{\partial \pi_i} = 0$$

both for $i = 1, .., n$. Dividing the above equations by $\frac{\partial e_i}{\partial s_i}$ and $\frac{\partial e_i}{\partial \pi_i}$ respectively, we obtain:

$$c_i'(e_i) + \beta_i \pi_i f'(e_i - s_i)\left(\frac{\partial e_i/\partial s_i - 1}{\partial e_i/\partial s_i}\right) = -\lambda_1 \tag{10}$$

$$c_i'(e_i) + \frac{\mu_i}{\partial e_i/\partial \pi_i} + \frac{\beta_i f(e_i - s_i)}{\partial e_i/\partial \pi_i} + \beta \pi_i f'(e_i - s_i) = -\lambda_1 \tag{11}$$

for all $i = 1, ..., n$. These can be used to characterize the cost-minimizing program to control emissions with standards when it is cost-effective to allow non-compliance. This is done in Proposition 2 below.

Proposition 2 *If the optimal policy* $(\pi_1^*, \pi_2^*, ...\pi_n^*, s_1^*, s_2^*, ...s_n^*)$ *allows non compliance for all firms, it is characterized by:*

$$c_i'(e_i) + \beta_i \pi_i^* f'(e_i - s_i^*)\left(\frac{\partial e_i/\partial s_i - 1}{\partial e_i/\partial s_i}\right) = c_j'(e_j) + \beta_j \pi_j^* f'(e_j - s_j^*)\left(\frac{\partial e_j/\partial s_j - 1}{\partial e_j/\partial s_j}\right) \tag{12}$$

$$c_i'(e_i) + \frac{\mu_i}{\partial e_i/\partial \pi_i} + \frac{\beta_i f(e_i - s_i^*)}{\partial e_i/\partial \pi_i} + \beta \pi_i^* f'(e_i - s_i^*) = c_j'(e_j) + \frac{\mu_j}{\partial e_j/\partial \pi_j} + \frac{\beta_j f(e_j - s_j^*)}{\partial e_j/\partial \pi_j} + \beta \pi_j^* f'(e_j - s_j^*) \tag{13}$$

for all $i \neq j$, $(i, j) = 1, ..., n$.

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strategic behaviour on the part of the firms in a dynamic context and, under these circumstances, the credibility and political support of the program may erode over time. However, we want to emphasise that, as we show latter in the paper, when the regulator can choose the emission standard, monitoring effort, and the structure of the penalty, the optimal design of the policy must induce perfect compliance. Therefore, the potential perverse dynamic effects of allowing non-compliance are avoided in the cost-effective design of the programs.

Proof of Proposition 2: it follows from the previous discussion.

Proposition 2 tells that when it is cost-effective to allow non-compliance for every firm, the regulator has to choose π_i and s_i such that: (a) the sum of the expected marginal abatement plus sanctioning costs of moving s_i is the same across firms (from (12)), and (b) the sum of the expected marginal abatement, monitoring, and sanctioning costs of changing π_i is the same across firms (from (13)). Condition (12) is quite intuitive. The firm reacts to a change in s_i by adjusting e_i by the amount $\partial e_i/\partial s_i$, in expected terms. This change in e_i has an effect on the abatement costs of firm i, but also an effect on the sanctioning costs of the regulator. We know that $0 < \partial e_i/\partial s_i < 1$.[6] Thus, a change in s_i causes the level of violation to change and, therefore, the level of the expected fines with which the regulator is going to charge firm i. This, in turn, means a change in the expected sanctioning costs for the regulator. The regulator sets s_i, equating these two marginal costs among firms. It does a similar thing when adjusting π_i (condition 13). A marginal change in the inspection probability affects all costs of the program: it affects the abatement costs of firm i *via* a change in the level of emissions; it affects the auditing costs directly; and it affects the sanctioning costs because it changes the number of violations being discovered and the amount of violation by firm i. The regulator sets π_i^* such that the sum of these three marginal costs, measured in units of emissions, are the same among all firms.

Furthermore, from (10) and (11), we can obtain the following:

$$\frac{\mu_i}{\partial e_i/\partial \pi_i} + \frac{\beta_i f(e_i - s_i^*)}{\partial e_i/\partial \pi_i} = -\frac{\beta_i \pi_i^* f'(e_i - s_i^*)}{\partial e_i/\partial s_i} \tag{14}$$

for all $i = 1, ..., n$. This condition says that, in the cost-minimizing solution, the regulator equates the marginal costs of moving the standard with that of moving the monitoring probability for every firm. More specifically, the sum of the marginal monitoring and sanctioning costs of moving π_i is equal to the marginal sanctioning costs of moving s_i for every firm i.

[6] This result was obtained as part of the Proof of Proposition 1; see Appendix for details.

We can conclude from Proposition 2 that the cost-effective level of emission standards are firm-specific whenever abatement and/or enforcement costs differ among firms. Assuming μ_i and β_i to be the same for all firms, condition (8) either holds or not *for every firm*. Thus, the regulator must induce compliance or non-compliance for every firm in the program. In this case, it would be the heterogeneity in marginal abatement costs $c_i'(e_i)$ that would call for firm-specific standards. Similarly, if marginal abatement costs are the same for all firms, but monitoring and sanctioning costs differ among firms ($\mu_i \neq \mu_{j,}$ $\beta_i \neq \beta_j$) the cost-minimizing standards could also differ among firms.

Finally, in the case in which monitoring and sanctioning costs differ between firms and condition (8) holds for one group of firms but not for another, the conditions characterizing the expected-cost-minimizing design of the program would be given by (9) , for the group of firms for which condition (8) holds plus conditions (12) and (13) for the group of firms for which it does not hold.

3.2 The regulator can choose the structure of the penalty function

Having characterized the optimal program when it is optimum to induce compliance and when it is not, we now allow the regulator to choose the structure of the penalty function f and, therefore, the optimality of inducing compliance or not. We consider only two marginal fine structures: linear and increasing. The general fine structure can be written as $f(e-s) = \phi(e-s) + \frac{\gamma}{2}(e-s)^2$, where ϕ is a positive constant and $\gamma \geq 0$. Consequently, the regulator has basically to compare four possible alternatives and choose the one that minimizes the total costs of reaching the cap T on emissions. The four alternatives are: (1) to induce compliance with linear penalties, (2) to induce compliance with increasing penalties, (3) to allow violations with linear penalties, and (4) to allow violations with increasing penalties. To induce compliance with linear or increasing penalties has the same minimum costs because under compliance there are no sanctioning costs. Also, to allow non-compliance with linear penalties is ruled out by Proposition 1: it is never cost-effective to allow non-compliance when the marginal fine is linear. Therefore, the choice for the regulator boils down to a comparison between the costs of two alternatives: to induce compliance with a linear or increasing marginal penalty, or to allow violations with increasing penalties. The result of this comparison is given in the next Proposition:

Proposition 3 *The optimal policy* $(s_1^*, s_2^*, ..., s_n^*, \pi_1^*, \pi_2^*, ...\pi_n^*, f^*)$ *induces compliance and it is characterized by (1)* $c_i'(s_i^*)+\mu_i \frac{d\pi_i^*}{ds_i} = c_j'(s_j^*)+\mu_j \frac{d\pi_j^*}{ds_j}$ *for all* $i, j = 1, ..., n$, $i \neq j$, *(2)* $\pi_i^* = \frac{-c_i'(s_i^*)}{f'(0)}$ for all $i = 1, .., n$, *and (3)* $f^* = \phi(e_i - s_i) + \frac{\gamma}{2}(e_i - s_i)^2$ *for all* i, *with* ϕ *set as high as possible and* $0 \leq \gamma \leq \min \left[\frac{\beta_i}{\mu_i}\right] \times \phi^2$.

Proof of Proposition 3: see the Appendix.

The cost-minimizing policy when a regulator wants to cap aggregate emissions of a given pollutant to a certain level T through emission standards will be one that induces compliance with a constant marginal penalty or an increasing marginal penalty, as long as $\mu_i \gamma \leq \beta_i \phi^2$ for all i (otherwise, the regulator mistakenly increases the cost of the program by making it cost-effective to allow violations). Because there are no sanctioning costs in equilibrium, the penalty structure affects the program's costs only through the monitoring costs: the larger the value of ϕ $(f'(0))$, the lower π_i^*, for all i. Nevertheless, precisely because γ does not affect π_i^*, a penalty function with a positive value of γ such that $0 \leq \gamma \leq \min \left[\frac{\beta_i}{\mu_i}\right] \times \phi^2$ is also optimum because it satisfies (8) and does not affect the minimum costs of the program. Our conclusion in this respect differ from that in Arguedas (2008).

Proposition 3 has important implications for the real-world policy design. The first and most obvious one is that there is no justification in terms of the costs of the program to design it to allow violations if the fine structure is under the control of the environmental policy administrator. If this is not the case and, for example, it is the legislature that sets the standards and another agency or office is in charge of designing the monitoring and enforcing strategy, for which it uses fine structures defined by the general civil or criminal law, the resulting regulatory design will probably be sub-optimal, except for the cases in which the penalty structure is appropriate to induce perfect compliance and the offices are coordinated so as to set standards and monitoring probabilities according to this proposition.

Proposition 3 does not give a clear rule for setting ϕ "as high as possible". In the real world, ϕ will have an upper boundary determined by things such as the possibility that firms have insufficient assets to cover the fines (Segerson and Tietenberg 1991) or the unwillingness of judges or juries to impose very high penalties (Becker 1968). Note that if this upper boundary of ϕ is combined with a binding monitoring budget, the environmental regulator may not be capable of assuring compliance for all i and, hence, to minimize the total costs of the emissions control program.

4 Comparing costs of emission standards and tradable permits

4.1 Optimally designed programs

We have seen that the optimal design of a program based on emissions standards is one in which standards are firm-specific (set according to Proposition 3) and perfectly enforced with a fine structure that can be linear or increasing in the margin, as long as ϕ is set as high as possible and condition (8) holds. We know from Stranlund (2007) that the optimal design of a program based on tradable permits is one in which the program is perfectly enforced, where every firm is audited with a homogeneous probability $\pi^* = \frac{p^*}{\phi}$ for all i, with p^* being the full-compliance equilibrium price of the permits market (i.e.: the price of the permits that makes the aggregate demand for permits equal to the supply of permits, when the aggregate supply of permits L is equal to the target T) and $\phi = f'(0)$. We can conclude from this result that, as in the case of emission standards, the structure of the penalty function (whether it is increasing at a constant or an increasing rate) does not affect the equilibrium (minimum) costs of the program, as long as $\mu_i \gamma \leq \beta_i \phi^2$ (it is cost-effective to induce perfect compliance). What affects the program's cost is ϕ. The question remains whether a regulator interested in controlling emissions of a given pollutant by setting a cap on aggregate emissions in a cost-minimizing manner should implement a perfectly enforced program based on firm-specific standards as in Proposition 3 above or a perfectly enforced program based on tradable permits as in Stranlund (2007). That is, once we know the optimal design of the programs based on the two instruments, which instrument should a regulator use if it wants to minimize the total costs of the program? The answer is given in the following Proposition:

Proposition 4 *A regulator that wants to cap the aggregate level of emissions of a given pollutant from a set of firms will minimize the total costs of doing so by implementing firm-specific emissions standards and perfectly enforcing this program according to Proposition 3. A system of tradable permits minimizes the total costs of such a pollution control program only if* $\mu_i = \mu_j$ *for all* $i \neq j$, $(i, j) = 1, .., n$, *or* $f''(0) = 0$.

Proof of Proposition 4: The proof that the total costs of an emission standards program are lower than the total costs of a transferable emission permits system is trivial. By definition, in the optimally designed emission standards program, which has to induce perfect compliance, the emission responsibilities (standards) and monitoring probabilities are allocated so as to minimize the total costs of a program that caps aggregate emissions at T. Therefore, the total costs of the emission standards program must be lower than the total costs of an optimally designed program based on tradable permits, which produces a different allocation of emissions and monitoring probabilities. Put differently, an optimally designed tradable permits program does not minimize the total costs of capping aggregate emissions at a certain level T, unless the monitoring costs do not differ between firms or the marginal penalty is constant. We provide a proof of this latter assertion below.

In order to make the regulator's problem under a system of tradable permits comparable to the regulator's problem under a system of emission standards, assume that under a system of tradable permits, a cost-minimizing regulator chooses the level of violation v_i and the level of monitoring π_i for each firm i, $i = 1, ..., n$, where $v_i = e_i - l_i$, and l_i is the quantity of permits demanded by firm i. More formally, the regulator's problem is:

$$\min_{\substack{(v_1,...,v_n)\\(\pi_1,...,\pi_n)}} \sum_{i=1}^{n} c_i\left(v_i + l_i\left(p^*, \pi_i\right)\right) + \sum_{i=1}^{n} \mu_i \pi_i + \sum_{i=1}^{n} \pi_i \beta_i f(v_i)$$

subject to

$$\sum_{i=1}^{n} v_i + l_i\left(p^*, \pi_i\right) = T$$

and

$$v_i \geq 0 \text{ for all } i = 1.., n.$$

where $l_i\left(p^*, \pi_i\right)$ is the demand of firm i for permits and p^* is the full compliance equilibrium price of permits, as already defined ($i.e$: the solution to $\sum_{i=1}^{n} l_i\left(p^*, \pi_i\right) \equiv L = T$, L being the total number of permits issued).We know from Stranlund and Dhanda (1999) that, independently of its compliance status, in a competitive permits market, every firm i decides its level of emissions such that $-c_i'(\cdot) = p$. Therefore, the optimal choice of emissions can be written as a function of the permit price: $e_i = e_i(p)$. We also know from Stranlund and Dhanda (1999) that every non-compliant firm is demanding permits so that $p = \pi_i f'(v_i = e_i(p) - l_i)$. But given that cost-effective monitoring requires $\pi_i = p/f'(0)$ for all $i = 1, .., n$, it is also true that $p = \pi_i f'(v_i)$ holds when $v_i = 0$. So this equation gives the firm's demand for permits $l_i\left(p, \pi_i\right)$, for $v_i \geq 0$.

The Lagrangian of this problem is:

$$\Lambda = \sum_{i=1}^{n} c_i\left(v_i + l_i\left(p^*, \pi_i\right)\right) + \sum_{i=1}^{n} \mu_i \pi_i + \sum_{i=1}^{n} \pi_i \beta_i f(v_i) + \lambda\left(\sum_{i=1}^{n} v_i + l_i\left(p^*, \pi_i\right) - T\right)$$

The Kuhn-Tucker conditions of this problem are:

$$\frac{\partial\Lambda}{\partial\pi_i} = c_i'(\cdot)\left(\frac{\partial l_i}{\partial p^*}\frac{\partial p^*}{\partial\pi_i} + \frac{\partial l_i}{\partial\pi_i}\right) + \mu_i + \beta_i f(v_i) + \lambda\left(\frac{\partial l_i}{\partial p^*}\frac{\partial p^*}{\partial\pi_i} + \frac{\partial l_i}{\partial\pi_i}\right) \geq 0; \qquad (15)$$

$$\pi_i \geq 0; \frac{\partial\Lambda}{\partial\pi_i}\pi_i = 0,\ i = 1, .., n$$

$$\frac{\partial\Lambda}{\partial v_i} = c_i'(\cdot) + \pi_i\beta_i f'(v_i) + \lambda \geq 0; v_i \geq 0; \frac{\partial\Lambda}{\partial v_i}v_i = 0,\ \ i = 1, .., n \qquad (16)$$

$$\frac{\partial\Lambda}{\partial\lambda} = \sum_{i=1}^{n} v_i + l_i\left(p^*, \pi_i\right) - T = 0$$

When it is optimum to induce perfect compliance for all i ($v_i = 0$), assuming $\pi_i > 0$ for all i, (15) can be re-written as:

$$\frac{\partial\Lambda}{\partial\pi_i} = c_i'(\cdot) + \frac{\mu_i}{\frac{\partial l_i}{\partial p^*}\frac{\partial p^*}{\partial\pi_i} + \frac{\partial l_i}{\partial\pi_i}} + \lambda = 0;\ \ i = 1, .., n \qquad (17)$$

Using $-c_i'(\cdot) = p$ (from the firm's optimal choice of emissions) and assuming $\frac{\partial p^*}{\partial\pi_i} = 0$ (perfect competition in the permits market), (17) can be written as:

$$-p^* + \frac{\mu_i}{\partial l_i/\partial\pi_i} = -\lambda \text{ for all } i = 1, .., n$$

This implies that the following identity must hold in the cost-minimizing design of a perfectly enforced tradable permits market: $-p^* + \frac{\mu_i}{\partial l_i/\partial\pi_i} = -p^* + \frac{\mu_j}{\partial l_j/\partial\pi_j}$ for all $i \neq j, (i, j) = 1, .., n$. Now, using $p = \pi_i f'(v_i)$, $\frac{\partial l_i}{\partial\pi_i} = \frac{f'(v_i)}{\pi_i f''(v_i)}$ for all $i = 1, .., n$. So, when $v_i = 0$, we can write $-p^* + \mu_i\frac{\pi_i f''(0)}{f'(0)} = -p^* + \mu_j\frac{\pi_j f''(0)}{f'(0)}$ for all $i \neq j, (i, j) = 1, .., n$. Substituting π_i for $p^*/f'(0)$ for π_i and π_j :

$$-p^* + \mu_i\frac{p^* f''(0)}{\left(f'(0)\right)^2} = -p^* + \mu_j\frac{p^* f''(0)}{\left(f'(0)\right)^2} \text{ for all } i \neq j, (i, j) = 1, .., n$$

In a competitive market for emission permits (i.e.: one that generates a unique equilibrium price p^*), the above equality holds if and only if $\mu_i = \mu_j$ or if $f''(0) = 0$. Thus, we can conclude that if $f''(0) \neq 0$ and $\mu_i \neq \mu_j$ for any two firms i and j, $i \neq j$, a competitive system of tradable permits will not minimize the total costs of a program that caps aggregate emissions at a certain level, Q.E.D.

We end this subsection by comparing our Proposition 4 with Chávez et al. (2009). These authors conclude that a program based on tradable permits is less or equally costly than a program based on emission standards. The difference with what we state in Proposition 4 lies in that these authors assume that the regulator has imperfect information about the abatement costs of the regulated firms, whereas our Proposition 4 relies on the assumption of perfect information of the regulator with regard to the abatement costs of the firms. Under the assumption of imperfect information, the regulator in Chávez et al. (2009) can only assure perfect compliance by monitoring all firms as if they had the largest possible marginal abatement costs (regardless of how the standards are set). In this case, the regulator always (1) monitor some firms more frequently than needed, and (2) cannot assign emissions responsibilities (set the emission standards) in an abatement-cost minimizing way. Consequently, the system based on emissions standards is more or equally expensive than the system based on transferable permits, as concluded by Chávez et al. (2009). We remove the assumption of perfect information in section 5.

4.2 Comparing costs when it is cost-effective to allow non-compliance

As discussed above, it may be a common situation in the real world that the fine structure is outside the control of the environmental authority. Assume that this is the case and that $\gamma > 0$. In this setting, whether the regulator has to perfectly enforce the program or not depends on the relative size of the monitoring and sanctioning parameters (i.e.: whether $\mu_i\gamma \leq \beta_i\phi^2$ for all i or not). Assume that $\mu_i\gamma > \beta_i\phi^2$ for all i. Then it is cost-effective to design a program that allows a given level of non-compliance for all i. In this case, how do the costs of a program based on emission standards compare with those of one based on tradable permits?

In order to answer this question, we first characterize the cost-effective design of a pollution-capping program based on tradable permits when it is cost-effective to allow a given level of aggregate non-compliance. Then we see if this optimally designed program minimizes the total costs of reaching the cap T.

4.2.1 Characterization of the cost-effective design of a program based on tradable permits when it is cost-effective to allow non-compliance

When it is optimum not to induce perfect compliance for all i ($v_i > 0$ for all i), equations (15) and (16) can be re-written, assuming $\pi_i > 0$ for all i, as:

$$\frac{\partial \Lambda}{\partial \pi_i} = c_i'(\cdot) + \frac{\mu_i + \beta_i f(v_i)}{\frac{\partial l_i}{\partial p^*}\frac{\partial p^*}{\partial \pi_i} + \frac{\partial l_i}{\partial \pi_i}} + \lambda = 0;\ i = 1, .., n \tag{18}$$

$$\frac{\partial \Lambda}{\partial v_i} = c_i'(\cdot) + \pi_i \beta_i f'(v_i) + \lambda = 0,\ i = 1, .., n \tag{19}$$

These equations characterize the optimal design of a tradable permits program when it is cost-effective to allow all firms to violate their permit holdings ($e_i - l_i > 0$). In a fashion similar to that of the emission standards program, in the optimally designed tradable permits program, the regulator sets π_i and v_i for all i such that: (a) the sum of the marginal abatement, monitoring, and sanctioning costs of changing π_i are equal across firms (equation 18) and (b) the sum of marginal abatement and sanctioning costs of changing v_i are equal across firms (equation 19). From equations (18) and (19) we can also obtain:

$$\frac{\mu_i + \beta_i f(v_i)}{\frac{\partial l_i}{\partial p^*}\frac{\partial p^*}{\partial \pi_i} + \frac{\partial l_i}{\partial \pi_i}} = \pi_i \beta_i f'(v_i),\ i = 1, .., n \tag{20}$$

Therefore, in the optimal design of a tradable permits program when it is cost-effective to allow all firms to violate their permit holdings, the regulator sets the sum of the marginal monitoring and sanctioning costs of changing π_i equal to the marginal sanctioning costs of moving v_i for every firm i.

4.2.2 Comparison of Costs

Having characterized the optimal emissions trading program, we now show that this program minimizes the total costs of capping aggregate emissions to T only under even more special conditions than in the case of perfect enforcement. In order to do this, we recall from the proof of Proposition 4 that every firm i that violates its permit holdings in a competitive emission permits market chooses its level of emissions such that $-c_i'(\cdot) = p^*$ and the quantity of permits to demand such that $p^* = \pi_i f'(v_i)$. Using both expressions, we can write (19) as:

$$(-1+\beta_i)\, p^* = -\lambda, \text{ for all } i = 1,..,n$$

or

$$\beta_i = 1 - \frac{\lambda}{p^*}, \text{ for all } i = 1,..,n$$

It is clear from the above equation that if sanctioning costs differ among firms ($\beta_i \neq \beta_j$ for some $i \neq j$, $(i,j) = 1,..,n$), a competitive permits market (one that generates a unique equilibrium price p^* for all firms) will not minimize the total costs of capping aggregate emissions to a level T, while allowing some degree of noncompliance.

Moreover, $\beta_i = \beta_j$ is a sufficient but not a necessary condition for this result to hold. If $\beta_i = \beta_j$ for all $i \neq j$ but $\mu_i \neq \mu_j$ for some (i,j), and we assume that the permits market is perfectly competitive, so that $\frac{\partial p^*}{\partial \pi_i} = 0$, then equation (20) can be written as:

$$\frac{\mu_i + \beta f(v_i)}{\partial l_i / \partial \pi_i} = \pi_i \beta f'(v_i) \text{ for all } i = 1,..,n$$

Using $\partial l_i / \partial \pi = f'(v)/\pi f''(v)$,

$$(\mu_i + \beta f(v_i)) \frac{f''(v_i)}{(f'(v_i))^2} = \beta \text{ for all } i = 1,..,n$$

This condition will not be met except in the special case in which $\mu_i = 0$ for all i, and $f(v_i)\frac{f''(v_i)}{(f'(v_i))^2} = 1$. Therefore, it is only under costless monitoring, equal sanctioning costs between firms, and $f(v_i)\frac{f''(v_i)}{(f'(v_i))^2} = 1$ that a system of tradable emission permits will minimize the costs of capping aggregate emissions when it is cost-effective to allow violations.

We express this result more formally in the Proposition below.

Proposition 5 *If a regulator wants to set a cap on the aggregate level of emissions of a pollutant and it is cost-effective to allow all firms to violate the regulation* ($\mu_i \gamma > \beta_i \phi^2$ *for all* i)*, the regulator will minimize the total costs of such a regulatory program by implementing a system of firm-specific emissions standards as characterized by Proposition 2.*

Proof of Proposition 5: It follows from the previous discussion.

4.3 Concluding comments to Section 4

Proposition 4 states that in order to minimize the total costs of a pollution control program that caps aggregate emissions to a certain level, an environmental regulator with control over the penalty structure should choose this structure so as to induce perfect compliance and implement a program based on firm-specific emissions standards designed according to Proposition 3. A system of tradable permits would always be more costly than the latter unless the regulator chooses a flat marginal penalty. Proposition 5 tells that when the fine structure is outside the control of the environmental regulator and it is cost-effective to allow the firms to violate, the regulator should choose emission standards designed according to Proposition 2.

This relative cost-effectiveness of emission standards over tradable permits in both cases seems to contradict what environmental economists have been advocating for over the last forty years. In this respect, it should be pointed out, first, that the monitoring and enforcement costs were not taken into account in the analysis that led to the policy recommendation stating the superiority of tradable permits over emission standards. Only aggregate abatement costs, which tradable permits certainly minimize, were taken into account. But when enforcement costs are brought into the picture, tradable permits cannot always exploit the differences in abatement *and* monitoring costs. Also, environmental economists have been advocating tradable permits as a cost-effective policy instrument when compared to *uniform* (i.e.: not firm-specific) emission standards. Third, and perhaps more important, Propositions

4 and 5 build on the assumption that the regulator has the necessary information to design the program based on emission standards according to Proposition 3 and 2, respectively. This information is basically concerned with the abatement costs of the firms. Of course, in the real world, the regulator cannot perfectly observe the firms' marginal abatement costs. The regulator may, therefore, make mistakes when setting emission standards. If this is the case, the realized social costs of setting and enforcing a global cap on emissions via firm-specific standards could end up being more expensive than doing it via an emissions trading scheme. In fact, as we show in the next section, it is only under a system of tradable permits and a flat marginal penalty that the regulator can overcome the informational problem and attain cost-effectiveness.

5 Imperfect information

In this section, we first derive the condition under which it is cost-effective to induce compliance when the regulator has imperfect information on abatement costs and emissions are capped with standards. Second, we discuss in a less formal manner the impact of imperfect information on the decision of whether or not to induce compliance in the case of tradable permits and the choice of instruments.

5.1 The condition under which it is cost-effective to induce compliance with emissions standards

Contrary to Section 2, we now assume that the regulator has imperfect information about the abatement cost functions of the regulated firms. Given this, the regulator cannot predict with certainty with what level of emissions a specific firm will respond to a given pair (s_i, π_i). Consequently, the regulator's problem is now:

$$\min_{\substack{(s_1,s_2,..,s_n)\\(\pi_1,\pi_2,..,\pi_n)}} E\left[\sum_{i=1}^{n}\left(c_i(e_i,\theta_i)+\mu_i\pi_i+\beta_i\pi_i f(e_i-s_i)\right)\right]$$

subject to:

$$e_i = e_i(s_i, \pi_i, \theta_i)$$

$$E\left[\sum_{i=1}^{n} e_i\right] = T$$

$$s_i \leq E\left(e_i\right) \;\; \forall i = 1, ...n$$

where θ_i is known to the firm but not to the regulator, who treats it as a random variable, and $E\left[.\right]$ is the expectation operator. We assume that the regulator cannot observe the actual value of θ_i but knows all the possible values it can take. In particular, we assume that the regulator knows the maximum possible value that θ_i can take. We call θ_{J_i} this maximum possible value and π_{J_i} the minimum level of the monitoring probability that makes a firm of type θ_{J_i} comply with the standards s_i, given the penalty structure $(i.e : \pi_{J_i} = -c_i(s_i, \theta_{J_i})/f'(0))$. In this scenario, assuming as we did in Section 2 that the penalty structure is given to the regulator, it is possible to derive a new condition that tells whether it is expected-cost-effective to induce compliance when the regulator does not have perfect information on the abatement cost of each regulated polluter and caps the aggregate level of emissions using emissions standards. This new condition is stated formally in the following Proposition.

Proposition 6 *When the regulator does not have perfect information on the abatement costs of the firms and the penalty structure is given, the expected-cost-effective design of a pollution-control program that caps the aggregate level of emissions using standards calls the regulator to induce all firms to comply with the stan-*

dards if and only if:

$$\left(\frac{P(\theta_i < \theta_{J_i}) + P(\theta_i = \theta_{J_i}) \times \frac{\pi_{J_i} f''(0)}{c_i''(s_i,\theta_{J_i})+\pi_{J_i} f''(0)}}{P(\theta_i = \theta_{J_i}) \times \frac{f'(0)}{c_i''(s_i,\theta_{J_i})+\pi_{J_i} f''(0)}}\right) \times \left(\mu_i + Cov\left[c_i'(s_i,\theta_i), \frac{\partial e_i(s_i,\pi_{J_i},\theta_i)}{\partial \pi_i}\right]\right) \tag{22}$$

$$\leq \beta_i \pi_{J_i} f'(0) - Cov\left[c_i'(s_i,\theta_i), \frac{\partial e_i(s_i,\pi_{J_i},\theta_i)}{\partial s_i}\right] \text{ for all } i,$$

where $P(.)$ indicates the probability that θ_i takes the denoted values. If this condition is not met and the regulator wants to achieve the cap cost-effectively, it should allow those plants for which this condition is not met to violate the emission standards.

Proof of Proposition 6: see Appendix.

Note that this condition differs from condition (8) in three terms: the first parenthesis on the left-hand side of the inequality and the covariances $Cov\left[c_i'(s_i,\theta_i), \frac{\partial e_i(s_i,\pi_{J_i},\theta_i)}{\partial \pi_i}\right]$ and $Cov\left[c_i'(s_i,\theta_i), \frac{\partial e_i(s_i,\pi_{J_i},\theta_i)}{\partial s_i}\right]$. The first parenthesis is $E\left[\frac{\partial e_i(s_i,\pi_{J_i},\theta_i)}{\partial s_i}\right]/E\left[\frac{\partial e_i(s_i,\pi_{J_i},\theta_i)}{\partial \pi_i}\right]$ evaluated at $e_i(s_i,\pi_{J_i},\theta_i) = s_i$, as derived in the appendix. This is the expected change in π_i that is needed to keep the expected level of emissions constant when the standard s_i is decreased in the margin. It is the equivalent to the term $\pi_i f''(0)/f'(0)$ on the left-hand side of (25) that was derived for the case of perfect information. Therefore, the expected decline in monitoring costs that can be attained by marginally decreasing the standard is comprised of a first term that, similar to the case of perfect information, captures the expected change in π_i times the cost of an inspection, μ_i. Nevertheless, imperfect information adds a component of abatement costs to the costs of moving π_i such that e_i remains constant in expected terms. These are captured by the covariance $Cov\left[c_i'(s_i,\theta_i), \frac{\partial e_i(s_i,\pi_{J_i},\theta_i)}{\partial \pi_i}\right]$, which is positive. (See Appendix). The right-hand side of the inequality (22) differs with respect to the right-hand side of (8) in a covariance term that, similarly to the covariance term on the left-hand side, captures the uncertainty of the regulator with respect to the change in abatement costs when decreasing the standard in the margin. This covariance is positive (See appendix), and its presence obeys the fact that there is a chance that the firm will not violate the new lower standard, but instead will decrease its level of emissions in the same quantity. Therefore, the expected level of the marginal sanctioning costs is lower than the certain level in the case of perfect information.

It is also worth noting that, unlike the case of perfect information, the regulator does not necessarily have to induce perfect compliance with the emission standards if it uses a flat marginal penalty in the case of imperfect information. Setting $f''(0) = 0$ in (22) does not produce a certain inequality, as in the case of perfect information.

5.2 The choice of policy instruments under incomplete information: a discussion

In the previous section, we derived the condition under which it is cost-effective to induce perfect compliance in a system of emission standards when the regulator has imperfect information on abatement costs. It is outside the scope of this paper to reproduce all the analysis performed for the case of perfect information in the case of imperfect information. Nevertheless, we think that it may be useful to end the paper with a less formal discussion regarding the impact of imperfect information on the decision regarding whether to induce compliance or not in the case of tradable permits and the consequences for the choice of instruments.

In the classical environmental economics literature of perfect and costless enforcement, an important difference of tradable permits and emission standards is that with the former the regulator needs to know nothing about abatement costs. All the regulator needs to do is set the desired cap T and issue an equal number of permits. Then the market would assign emissions responsibilities in a cost-effective manner through the price mechanism. In equilibrium, all firms would be emitting and buying permits up to the point where $-c_i'(e_i = l_i, \theta_i) = p^*$, with p^* defined as before, and also equal to the aggregate marginal cost $C'(.)$ evaluated at T (i.e.: $p^* = C'(T)$).

Quite differently, in the case of costly enforcement, whether or not it is true that a total cost-minimizing regulator needs to know nothing about the abatement costs of the firms depends on the fine structure.

Recall from the case of perfect information, that the cost-effective design of a perfectly enforced program based on tradable permits is one in which every firm is audited with a homogeneous probability $\pi^* = \frac{p^*}{\phi}$, $\phi = f'(0)$. Therefore, in order to set the proper perfect-compliance-inducing inspection probability π^*, the regulator would apparently need to predict the perfect compliance equilibrium price of the permits p^*. Given that it is still true that $p^* = C'(T)$ in this case, this means that the regulator would need to know the aggregate marginal abatement cost function, which, in turn, requires knowing θ_i, for every i. Nevertheless, we know from Stranlund (2007) that this information problem can be overcome with a simple design of the marginal penalty. Stranlund proposes the marginal penalty to be constant and set such that $\phi = h \times p$, with $h > 1$ and p the on-going permit price. In this case, the required minimum level of the inspection probability to assure perfect compliance is $\pi = \frac{1}{h}$. In other words, inspecting the firms with a constant inspection probability and issuing a number of permits $L = T$, the regulator could attain a perfectly enforced program without the need to know the abatement costs of the sources.

Stranlund does not analyze the case of an increasing marginal penalty. Nevertheless, his recommendation with regard to the design of the fine structure is also applicable to this case. The issue in this case is that, unlike the case of the linear marginal penalty, when $f''(0) \neq 0$, it is not always true that it is cost-effective to perfectly enforce the program. This means that the regulator would need to evaluate how the costs of the program change (with respect to the cost of perfectly enforcing it) when marginally decreasing the supply of permits L and varying π accordingly so that the total level of emissions remains constant (and equal to the target T). As before, the regulator could attain perfect compliance in this case by setting $f'(0) = h \times p$. But, could the regulator design the program if it wants to allow a certain level of non compliance without knowing the firms' abatement costs *a priori*? The answer is no. Recalling that if every firm faces the same price p^*, the same inspection probability π and the same fine structure, the level of the violation would be the same for all the firms, i.e.: $(T - L)/n$, a regulator that wants to implement a program that allows a certain level of aggregate violation $T - L$ has to issue a number of permits $L < T$ and inspect every firm with the probability π that makes $p^* = \pi \times f'((T - L)/n)$. But of course, in order to be able to do this the regulator has to be able to predict p^* first, which is not possible without knowing the firms' abatement costs a priori.

Summing up, if the marginal penalty is flat, the regulator knows it has to perfectly enforce the tradable discharge permits program and it can surmount the informational problem, attaining cost-effectiveness. In the case of a marginally increasing penalty, the regulator cannot know *a priori* whether it has to perfectly enforce the program or not because, in the latter case, it cannot surmount the informational problem. This may be another important reason to advocate the use of flat penalties together with tradable discharge permits: they eliminate the uncertainty of whether or not to induce full compliance, and they allow the regulator to set the perfect-compliance-inducing inspection probability knowing nothing about the abatement costs. In other words, tradable discharge permits with flat penalties tied to the permit price allow the regulator to implement the cost-effective design of the program in the case of imperfect information.

6 Conclusion

In this paper, we first derive the condition under which it is cost-effective for a regulator to induce perfect compliance in an emissions control program. This condition depends on the cost of monitoring and sanctioning firms, as well as on the structure of the penalty for violations. It is not instrument-dependent. If the condition is met, the regulator has to induce perfect compliance independently of whether it is implementing emission standards or transferable permits. Because we assume that the regulator's monitoring and sanctioning costs are firm-specific, the condition itself is firm-specific. In other words, it is possible that cost-effectiveness calls the regulator to induce some firms to comply with the legislation while at the same time letting others violate it. This cannot happen when one assumes that the regulator's monitoring and sanctioning costs are the same for all firms. In this case, the regulator either has to induce compliance on all firms or allow all firms to violate.

Second, we characterize the total-cost-minimizing design of a program that caps aggregate emissions of a given pollutant from a set of heterogeneous firms based on emissions standards when it is cost-effective to induce perfect compliance and when it is not. We then allow the regulator to choose the optimality of inducing compliance or not. Doing this, we find that the total cost-effective design of such a program is one in which standards are firm-specific and perfectly enforced.

Third, we compare the costs of such an optimally designed program with that of an optimally designed program based on a perfectly competitive emission permits market, which also calls for perfect enforcement according to Stranlund (2007). This comparison allows us to conclude that the total costs of the latter are always larger than the costs of the former, except when the regulator's cost of monitoring a firm's emissions are the same for all firms or the marginal penalty for violations is constant. Moreover, when it is cost-effective to allow violations, tradable permits minimize costs only under even more special conditions.

In deriving the above results, we assume that the regulator has perfect information on the firms' abatement costs. This assumption is, of course, unrealistic. For this reason, we also derive the condition under which it is expected-cost-effective for a regulator to induce perfect compliance in an emissions control program based on emission standards when it has imperfect information on abatement costs. This condition is different from the corresponding condition under complete information, depending on covariance and expectation terms that capture the fact that the regulator is uncertain with respect to the firms' reactions to the different pairs of emission standard and monitoring probability. Quite differently, in the case of tradable permits, the regulator could surmount the informational problem (and therefore the uncertainty of whether it has to induce perfect compliance or not in this program) using a constant marginal penalty tied to the observed price of the permits. On the contrary, the regulator cannot surmount the informational problem in the case of tradable permits if it uses an increasing marginal penalty. The policy recommendation that emerges from these results is clear: when capping emissions from a set of sources whose abatement costs are not perfectly known, environmental regulators should use tradable permits and perfectly enforce them with a constant marginal penalty tied to the permit price, if they want to minimize the total cost of the program.

Appendix

Proof of Proposition 1 If $e_i = s_i$, from (7) we know that $\lambda_2^i \geq 0$. Because we have also that $\lambda_1 \geq 0$, we can re-write the first-order conditions (4) and (5) of the regulator's problem as:

$$\begin{aligned}\frac{\partial L}{\partial s_i} &= \left\{c_i'(s_i) + \beta_i \pi_i f'(0) + \left(\lambda_1 - \lambda_2^i\right)\right\} \frac{\partial e_i}{\partial s_i} - \beta_i \pi_i f'(0) + \lambda_2^i = 0 \\ \frac{\partial L}{\partial \pi_i} &= \left\{c_i'(s_i) + \beta_i \pi_i f'(0) + \left(\lambda_1 - \lambda_2^i\right)\right\} \frac{\partial e_i}{\partial \pi_i} + \mu_i = 0\end{aligned}$$

Re-arranging the expressions and dividing:

$$\frac{\partial e_i / \partial s_i}{\partial e_i / \partial \pi_i} = \frac{\beta_i \pi_i f'(0) - \lambda_2^i}{-\mu_i}$$

From the firm's optimal choice of emissions, we know that:

$$-c_i'(e_i) = \pi_i f'(e_i - s_i)$$

From where,

$$\partial e_i / \partial \pi_i = \frac{-f'}{c_i'' + \pi_i f''} < 0 \tag{23}$$

and

$$0 < \partial e_i / \partial s_i = \frac{\pi_i f''}{c_i'' + \pi_i f''} < 1 \tag{24}$$

Because a cost-minimizing regulator that wants to achieve $e_i = s_i$ will set π_i such that $-c_i'(s_i) = \pi_i f'(0)$ in order not to waste monitoring resources, we can write:

$$\left.\frac{\partial e_i/\partial s_i}{\partial e_i/\partial \pi_i}\right|_{e_i=s_i} = \frac{\pi_i f''(0)}{c_i''(s_i)+\pi_i f''(0)} \times \frac{c_i''(s_i)+\pi_i f''(0)}{-f'(0)} = \frac{\pi_i f''(0)}{-f'(0)} = \frac{\beta_i \pi_i f'(0) - \lambda_2^i}{-\mu_i}$$

or

$$\mu_i \frac{\pi_i f''(0)}{f'(0)} = \pi_i \beta_i f'(0) - \lambda_2^i$$

From where, using $\lambda_2^i \geq 0$,

$$\mu_i \frac{\pi_i f''(0)}{f'(0)} \leq \pi_i \beta_i f'(0) \tag{25}$$

Dividing both sides of equation (25) by π_i, we obtain $\mu_i \frac{f''(0)}{f'(0)} \leq \beta_i f'(0)$ for all i. We have proven that when a cost-minimizing regulator induces perfect compliance, this condition is met. The reverse is also true. Assume to the contrary that $\mu_i \frac{f''(0)}{f'(0)} \leq \beta_i f'(0)$ holds but $e_i > s_i$. If $e_i > s_i$, we know from (7) that $\lambda_2^i = 0$ and

$$\begin{aligned}
\frac{\partial L}{\partial s_i} &= \left[c_i'(e_i) + \beta_i \pi_i f'(e_i - s_i) + \lambda_1\right] \frac{\partial e_i}{\partial s_i} - \beta_i \pi_i f'(e_i - s_i) = 0 \\
\frac{\partial L}{\partial \pi_i} &= \left\{c_i'(e_i) + \beta_i \pi_i f'(e_i - s_i) + \lambda_1\right\} \frac{\partial e_i}{\partial \pi_i} + \mu_i + \beta_i f(e_i - s_i) = 0
\end{aligned}$$

From these and the firm's optimal choice of emissions:

$$\frac{\partial e_i/\partial s_i}{\partial e_i/\partial \pi_i} = \frac{\pi_i f''(e_i - s_i)}{-f'(e_i - s_i)} = \frac{\beta_i \pi_i f'(e_i - s_i)}{-\mu_i - \beta_i f(e_i - s_i)}$$

After substituting for the functional form of f, operating, and rearranging, we can write:

$$\begin{aligned}
\mu_i \gamma - \beta_i \phi^2 &= \mu_i f''(0) - \beta_i \left(f'(0)\right)^2 \\
&= \beta_i \gamma \left(-\phi(e_i - s_i) - \frac{\gamma}{2}(e_i - s_i)^2 + 2\phi(e_i - s_i) + \gamma(e_i - s_i)^2\right) > 0
\end{aligned}$$

which is a contradiction. Hence, when $\mu_i \frac{f''(0)}{f'(0)} \leq \beta_i f'(0)$ is met, it is cost-effective for the regulator to induce firm i to comply with the emission standard, Q.E.D.

Proof of Proposition 3 In order to prove Proposition 3, we need first to answer a previous question: what is the cost-minimizing structure of the fine when it is optimum to induce compliance and when it is not?

If the optimal policy is going to induce compliance for all i, condition (8) requires that $\mu_i \gamma \leq \beta_i \phi^2$ for all $i = 1, ..., n$. We also know from Section 3 that, in this case, the characterization of the cost-effective design of a program based on standards calls $\pi_i^* = \frac{-c_i'(s_i^*)}{f'(0)} = \frac{-c_i'(s_i^*)}{\phi}$. From this, we can conclude that the regulator must choose the linear component ϕ of the fine structure as high as possible because this will decrease the optimum level of the inspection probability, π_i^*, and thereby the monitoring costs. Conceptually, this calls for $\phi = \infty$ because this will make the monitoring costs equal to zero, but in the real world there may be limits to the upper value of ϕ. If we call $\bar{\phi}$ the highest possible value of ϕ, any value of γ : $0 \leq \gamma \leq \min \left[\frac{\beta_i}{\mu_i}\right] \times \bar{\phi}^2$ will still make it cost-effective to induce compliance for every firm and will not have an effect on the minimum costs of the program, namely $\sum_{i=1}^n c_i(s_i^*) + \mu \sum_{i=1}^n \pi_i^*$.

Therefore, if the optimal policy induces compliance for all i, the cost-minimizing shape of the fine must be such that the linear component ϕ is set as high as possible. The value of the progressive component γ is irrelevant in equilibrium as long as $0 \leq \gamma \leq \min \left[\frac{\beta_i}{\mu_i}\right] \times \bar{\phi}^2$, where $\bar{\phi}$ is the chosen level of ϕ.

If the regulator is going to allow non-compliance, how does it have to choose ϕ and γ in order to minimize the costs of a program that produces T? In other words, can the regulator decrease the costs of the program by altering the fine structure (the value of ϕ and γ), once the optimal standards, inspections probabilities, and emissions have been chosen? Notice that to choose the appropriate fine structure, the regulator should choose the values of ϕ and γ keeping violations and fines constant. If $f(e-s) = \phi(e-s) + \frac{\gamma}{2}(e-s)^2$, changing ϕ and γ so as to keep f constant requires $\frac{e-s}{2} = -\frac{d\phi}{d\gamma}$. But with n firms, it is impossible to move ϕ and γ such that $\frac{e_i - s_i}{2} = -\frac{d\phi}{d\gamma}$ for all i. Keeping f constant for all i requires firm-specific fine parameters. We assume that this is the case, and we then show that the optimal design of the program calls for a uniform fine structure.

If the fine structure is firm-specific, we have $f_i(e_i - s_i) = \phi_i(e_i - s_i) + \frac{\gamma_i}{2}(e_i - s_i)^2$, and $f_i'(e_i - s_i) = \phi_i + \gamma_i(e_i - s_i)$ for each i. Now we ask how to choose ϕ_i and γ_i in order to minimize the costs of a program that produces T when it is optimal to allow violations. Following Arguedas (2008), we ask ourselves whether we can decrease the costs of a program that allows a certain level of violation for each firm by changing the fine structure (changing the values of ϕ_i and γ_i) while choosing $\pi_i = \pi_i^* = \frac{E\left[-c_i'(e_i)\right]}{f'(e_i - s_i)}$. In order to answer this question, we evaluate the Lagrangian of the regulator's problem at π_i^* when $e_i > s_i$ and $\sum_i e_i = T$ and change ϕ_i and γ_i such that $df_i = 0$, that is $-\frac{d\phi_i}{d\gamma_i} = \frac{e_i - s_i}{2}$.

$$L = \sum_{i=1}^{n} c_i(e_i) + \sum_{i=1}^{n} \mu_i \pi_i^* + \sum_{i=1}^{n} \beta_i \pi_i^* f_i(e_i - s_i)$$

$$dL = \frac{\partial L}{\partial \phi_i} d\phi_i + \frac{\partial L}{\partial \gamma_i} d\gamma_i$$

$$\begin{aligned} dL &= \left[\mu_i \frac{\partial \pi_i^*}{\partial \phi_i} + \beta_i \left[\frac{\partial \pi_i^*}{\partial \phi_i} f_i(e_i - s_i) + \pi_i^*(e_i - s_i)\right]\right] d\phi_i \\ &\quad + \left[\mu_i \frac{\partial \pi_i^*}{\partial \gamma_i} + \beta_i \left[\frac{\partial \pi_i^*}{\partial \gamma_i} f_i(e_i - s_i) + \pi_i^* \frac{(e_i - s_i)^2}{2}\right]\right] d\gamma_i \end{aligned}$$

Dividing both sides by $d\phi_i$ and substituting $\frac{d\gamma_i}{d\phi_i}$ for $-\frac{2}{e_i - s_i}$ we obtain

$$\begin{aligned} \frac{dL}{d\phi_i} &= \mu_i \frac{\partial \pi_i^*}{\partial \phi_i} + \beta_i \left[\frac{\partial \pi_i^*}{\partial \phi_i} \left(\phi_i(e_i - s_i) + \frac{\gamma_i}{2}(e_i - s_i)^2\right)\right] \\ &\quad - \frac{2\mu_i}{e_i - s_i} \frac{\partial \pi_i^*}{\partial \gamma_i} - \beta \left[\frac{\partial \pi_i^*}{\partial \gamma_i} \left(2\phi_i + \gamma_i(e_i - s_i)\right)\right] \end{aligned}$$

We know that $\frac{\partial \pi_i^*}{\partial \phi_i} = -\frac{-c_i'(e_i)}{\left[\phi_i + \gamma_i(e_i - s_i)\right]^2}$ and $\frac{\partial \pi_i^*}{\partial \gamma_i} = -\frac{-c_i'(e_i)}{\left[\phi_i + \gamma_i(e_i - s_i)\right]^2} \times (e_i - s_i)$. Substituting:

$$\begin{aligned} \frac{dL}{d\phi_i} &= -\frac{-c_i'(e_i)}{\left[\phi_i + \gamma_i(e_i - s_i)\right]^2} \left[\mu_i + \beta_i \left(\phi_i(e_i - s_i) + \frac{\gamma_i}{2}(e_i - s_i)^2\right)\right] \\ &\quad + \frac{-c_i'(e_i)}{\left[\phi_i + \gamma_i(e_i - s_i)\right]^2} \times (e_i - s_i) \left[\frac{2\mu_i}{e_i - s_i} + \beta_i \left(2\phi_i + \gamma_i(e_i - s_i)\right)\right] \end{aligned} \tag{26}$$

And after some operations, we obtain:

$$\frac{dL}{d\phi_i} = \frac{-c_i'(e_i)}{\left[\phi_i + \gamma_i(e_i - s_i)\right]^2}\left[\mu_i + \beta_i\left(\phi_i(e_i - s_i) + \frac{\gamma_i}{2}(e_i - s_i)^2\right)\right] > 0$$

This means that the regulator can decrease the costs of a program that allows a violation $(e_i - s_i)$ for each firm by decreasing ϕ_i (and increasing γ_i accordingly so as to keep the equilibrium fine constant).

Now, decreasing ϕ_i has a limit and this limit is $\phi_i = 0$. Under a negative value of ϕ_i, it will always exist a (sufficiently small) level of violation to make the fine negative. But a negative fine violates our assumption that $f \geq 0$ for all levels of violations. On the other hand, there is no theoretical maximum value for γ_i. In theory, this value is infinite and therefore, it is not firm-specific. Therefore, the cost-minimizing design of a program based on standards calls for a uniform penalty structure for all firms: $f(e_i - s_i) = \frac{\gamma}{2}(e_i - s_i)^2$ for all i. The regulator always decreases monitoring costs by increasing γ, for the same level of violation. This is true for all firms and therefore the regulator must set γ as high as possible for all firms. Because we are considering the case in which the regulator allows non-compliance, condition $\mu_i\gamma > \beta_i\phi^2$ for all $i = 1, ..., n$ must hold. And because we have just said that the cost-minimizing shape of the penalty function requires $\phi_i = 0$ for all $i = 1, ..., n$, the above condition only requires $\gamma > 0$. In conclusion, if the optimal policy allows non-compliance, the best shape of the penalty function is one in which the linear component is set equal to zero ($\phi = 0$), and the progressive component is set "as high as possible" for all firms.

Having determined what is the cost-minimizing structure of the fine when it is optimum to induce compliance and when it is not, we now prove Proposition 3. Following Arguedas (2008), we assume that it is optimum to allow non-compliance, and call the optimal policy $P^n = (s_1^n, s_2^n, ..., s_n^n, \pi_1^n, \pi_2^n, ...\pi_n^n, f^n)$, with $f^n = \frac{\gamma}{2}(e_i - s_i)^2$ for all i (with γ as high as possible following the results above), $\pi_i^n = \frac{-c_i'(e_i^n)}{\gamma(e_i^n - s_i^n)}$, and $\sum_{i=1}^n e_i^n = T$. Now consider an alternative policy $P^c = (s_1^c, s_2^c, ..., s_n^c, \pi_1^c, \pi_2^c, ...\pi_n^c, f^c)$ such that $s_i^c = e_i^n$ and $\pi_i^c = \pi_i^n$ for all i, and $f^c = \phi(e_i - s_i)$ for all i with

$\phi = \gamma \times \max_i \left[e_i^n - s_i^n\right]$. By construction, this policy induces compliance because $\pi_i^c f^{c\prime} = \pi_i^c \phi = \pi_i^c \gamma \times \max_i \left[e_i^n - s_i^n\right] \geq -c_i^{\prime}(e_i^n) = -c_i^{\prime}(s_i^c)$ for all i. Moreover, P^c is cheaper than P^n in terms because abatement costs are the same under both programs ($s_i^c = e_i^n$ for all i), monitoring costs are the same under both programs ($\pi_i^c = \pi_i^n$ for all i), but there are no sanctioning costs under policy P^c because there are no violations, **Q.E.D**.

Proof of Proposition 6 The Lagrange of the regulator's problem can be written as

$$
\begin{aligned}
L &= E\left[\sum_{i=1}^{n} \left(c_i(e_i(s_i,\pi_i,\theta_i),\theta_i) + \mu_i \pi_i + \beta_i \pi_i f(e_i(s_i,\pi_i,\theta_i) - s_i)\right)\right] \\
&\quad + \lambda_1 \left[E\left[\sum_{i=1}^{n} e_i(s_i,\pi_i,\theta_i)\right] - T\right] + \sum_{i=1}^{n} \lambda_2^i (s_i - E\left[e_i(s_i,\pi_i,\theta_i)\right])
\end{aligned}
$$

with λ_1 and λ_2^i being the $n+1$ multipliers. The $n \times 2 + n + 1$ necessary Kuhn-Tucker conditions for positive levels of the standard and the auditing probability are:

$$
\frac{\partial L}{\partial s_i} = E\left[c_i^{\prime}(e_i(s_i,\pi_i,\theta_i),\theta_i)\frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial s_i} + \beta_i \pi_i f^{\prime}(e_i(s_i,\pi_i,\theta_i) - s_i)(\frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial s_i} - 1)\right] \tag{27}
$$

$$
+\lambda_1 E\left(\frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial s_i}\right) + \lambda_2^i \left[1 - E\left(\frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial s_i}\right)\right] = 0, i = 1, ..., n
$$

$$
\frac{\partial L}{\partial \pi_i} = E\left[\begin{array}{c} c_i^{\prime}(e_i(s_i,\pi_i,\theta_i),\theta_i)\frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial \pi_i} + \mu_i \\ +\beta_i\left(f(e_i(s_i,\pi_i,\theta_i) - s_i) + \pi_i f^{\prime}(e_i(s_i,\pi_i,\theta_i) - s_i)\frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial \pi_i}\right)\end{array}\right] \tag{28}
$$

$$
+\lambda_1 E\left[\frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial \pi_i}\right] - \lambda_2^i E\left[\frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial \pi_i}\right] = 0, i = 1, ..., n
$$

$$
\frac{\partial L}{\partial \lambda_1} = E\left[\sum_{i=1}^{n} e_i(s_i,\pi_i,\theta_i)\right] - E = 0
$$

$$\frac{\partial L}{\partial \lambda_2^i} = s_i - E\left[e_i(s_i,\pi_i,\theta_i)\right] \leq 0, \lambda_2^i \geq 0, \lambda_2^i \times \left(s_i - E\left[e_i(s_i,\pi_i,\theta_i)\right]\right) = 0 \tag{29}$$

We assume that these conditions are necessary and sufficient to characterize the optimal solution of the problem.

If $E\left[e_i(s_i,\pi_i,\theta_i)\right] = s_i$, from (29) we know that $\lambda_2^i \geq 0$. Using the linearity of f', and operating we can re-write the Kuhn-Tucker conditions (27) and (28) as:

$$\begin{gathered}\left(E\left[c_i'(e_i(s_i,\pi_i,\theta_i),\theta_i)\right] + \beta_i\pi_i f'(0) + \lambda_1 - \lambda_2^i\right) \times E\left[\frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial s_i}\right] = \\ \beta_i\pi_i f'(0) - \lambda_2^i - Cov\left[c_i'(e_i(s_i,\pi_i,\theta_i),\theta_i), \frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial s_i}\right] \\ -\beta_i\pi_i\left(Cov\left[f'(e_i(s_i,\pi_i,\theta_i) - s_i), \frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial s_i}\right]\right), i = 1,...,n\end{gathered}$$

and

$$\begin{gathered}\left(E\left[c_i'(e_i(s_i,\pi_i,\theta_i),\theta_i)\right] + \beta_i\pi_i f'(0) + \lambda_1 - \lambda_2^i\right) \times E\left[\frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial \pi_i}\right] = \\ -\mu_i - \beta_i E\left[f(e_i(s_i,\pi_i,\theta_i) - s_i\right] - Cov\left[c_i'(e_i(s_i,\pi_i,\theta_i),\theta_i), \frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial \pi_i}\right] \\ -\beta_i\pi_i Cov\left[f'(e_i(s_i,\pi_i,\theta_i) - s_i), \frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial \pi_i}\right], i = 1,...,n\end{gathered}$$

Dividing both expressions, we obtain:

$$\left.\frac{E\left[\frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial s_i}\right]}{E\left[\frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial \pi_i}\right]}\right|_{E[e_i(s_i,\pi_i,\theta_i)]=s_i} = \tag{30}$$

$$\frac{\begin{gathered}+\beta_i\pi_i f'(0) - \lambda_2^i - Cov\left[c_i'(e_i(s_i,\pi_i,\theta_i),\theta_i), \frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial s_i}\right] \\ -\beta_i\pi_i\left(Cov\left[f'(e_i(s_i,\pi_i,\theta_i) - s_i), \frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial s_i}\right]\right)\end{gathered}}{\begin{gathered}-\mu_i - \beta_i E\left[f(e_i(s_i,\pi_i,\theta_i) - s_i\right] - Cov\left[c_i'(e_i(s_i,\pi_i,\theta_i),\theta_i), \frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial \pi_i}\right] \\ -\beta_i\pi_i Cov\left[f'(e_i(s_i,\pi_i,\theta_i) - s_i), \frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial \pi_i}\right]\end{gathered}}$$

From the firm's optimal choice of emissions, we know that:

$$-c_i'(e_i(s_i, \pi_i, \theta_i), \theta_i) \equiv \pi_i f'\left(e_i(s_i, \pi_i, \theta_i) - s_i\right)$$

if the firm violates the standard ($e_i(s_i, \pi_i, \theta_i) - s_i > 0$), and

$$-c_i'(s_i, \theta_i) \leq \pi_i f'(0)$$

if $e_i(s_i, \pi_i, \theta_i) = s_i$. From where:

$$\partial e_i(s_i, \pi_i, \theta_i)/\partial \pi_i = \frac{-f'\left(e_i(s_i, \pi_i, \theta_i) - s_i\right)}{c_i''(e_i(s_i, \pi_i, \theta_i), \theta_i) + \pi_i f''\left(e_i(s_i, \pi_i, \theta_i) - s_i\right)} < 0$$

and

$$0 < \partial e_i(s_i, \pi_i, \theta_i)/\partial s_i = \frac{\pi_i f''\left(e_i(s_i, \pi_i, \theta_i) - s_i\right)}{c_i''(e_i(s_i, \pi_i, \theta_i), \theta_i) + \pi_i f''\left(e_i(s_i, \pi_i, \theta_i) - s_i\right)} < 1$$

if $e_i(s_i, \pi_i, \theta_i) \geq s_i$, and

$$\partial e_i(s_i, \pi_i, \theta_i)/\partial \pi_i = 0$$

$$\partial e_i(s_i, \pi_i, \theta_i)/\partial s_i = 1$$

if $-c_i'(s_i, \theta_i) < \pi_i f'(0)$. From this analysis, we can write

$$E\left(\frac{\partial e_i(s_i, \pi_i, \theta_i)}{\partial \pi_i}\right) = P(\theta_i \leq \hat{\theta}_i) \times 0 + P(\theta_i > \hat{\theta}_i) \times \frac{-f'\left(e_i(s_i, \pi_i, \theta_i) - s_i\right)}{c_i''(e_i(s_i, \pi_i, \theta_i), \theta_i) + \pi_i f''\left(e_i(s_i, \pi_i, \theta_i) - s_i\right)}$$

where $\hat{\theta}_i$ is the maximum value of θ_i for which the firm is compliant with s_i, given π_i. That is $\hat{\theta}_i$ is such that $e_i(s_i, \pi_i, \hat{\theta}_i) \equiv s_i$. Therefore, $P(\theta_i \leq \hat{\theta}_i)$ can be interpreted as the probability that the firm i complies with the standard s_i, and $P(\theta_i > \hat{\theta}_i)$ the probability that the firm i violates the standard s_i. Incorporating the assumption of a quadratic fine structure, so that $f''(.) = \gamma$ for all $e_i \geq s_i$, we can rewrite the above expression as:

$$E\left(\frac{\partial e_i(s_i, \pi_i, \theta_i)}{\partial \pi_i}\right) = P(\theta_i > \hat{\theta}_i) \times \frac{-f'\left(e_i(s_i, \pi_i, \theta_i) - s_i\right)}{c_i''(e_i(s_i, \pi_i, \theta_i), \theta_i) + \pi_i f''(0)}$$

Similarly, we can also write:

$$E\left(\frac{\partial e_i(s_i, \pi_i, \theta_i)}{\partial s_i}\right) = P(\theta_i \leq \hat{\theta}_i) + P(\theta_i > \hat{\theta}_i) \times \frac{\pi_i f''(0)}{c_i''(e_i(s_i, \pi_i, \theta_i), \theta_i) + \pi_i f''(0)}$$

Using these two expressions we can write

$$\frac{E\left[\frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial s_i}\right]}{E\left[\frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial \pi_i}\right]} = \frac{P(\theta_i \leq \hat{\theta}_i) + P(\theta_i > \hat{\theta}_i) \times \frac{\pi_i f''(0)}{c_i''(e_i(s_i,\pi_i,\theta_i),\theta_i)+\pi_i f''(0)}}{P(\theta_i > \hat{\theta}_i) \times \frac{-f'(e_i(s_i,\pi_i,\theta_i)-s_i)}{c_i''(e_i(s_i,\pi_i,\theta_i),\theta_i)+\pi_i f''(0)}} \tag{31}$$

Now, in order to compare (31) with (30), we need to evaluate it at $E\left[e_i(s_i, \pi_i, \theta_i)\right] = s_i$. Assume that θ_i is a discrete random variable. Then:

$$E\left[e_i(s_i, \pi_i, \theta_i)\right] = \sum_{j=1}^{J_i} P(\theta_j) \times e_i(s_i, \pi_i, \theta_j)$$

where J_i are all the possible values of θ_i, ordered from the lowest to the largest, and $P(\theta_j)$ is the associated probability of each of these values. It is easy to see that $E\left[e_i(s_i, \pi_i, \theta_i)\right] > s_i$ for all $\pi_i < \pi_{J_i}$, with π_{J_i} denoting the value of the monitoring probability that makes firm i comply if $\theta_i = \theta_{J_i}$. In other words, $E\left[e_i(s_i, \pi_i, \theta_i)\right] = s_i$ if and only if $\pi_i \geq \pi_{J_i}$, i.e., the plant is monitored as if it had the highest possible marginal abatement costs. The proof of this result is quite intuitive. Assume that the regulator monitors firm i with a monitoring probability π_i such that $-c_i'(s_i, \theta_i) < \pi_i f'(0)$ for all possible values of θ_i, except for θ_{J_i}. In this case, $e_i(s_i, \pi_i, \theta_i) = s_i$ for all possible values of $\theta_i < \theta_{J_i}$, and $e_i(s_i, \pi_i, \theta_{J_i}) > s_i$. Given this, $E\left[e_i(s_i, \pi_i, \theta_i)\right] > s_i$. Therefore, $E\left[e_i(s_i, \pi_i, \theta_i)\right] = s_i$ requires $\pi_i \geq \pi_{J_i}$.

Assuming a cost-minimizing regulator, this will set $\pi_i = \pi_{J_i}$. But note that if the regulator sets $\pi_i = \pi_{J_i}$ it will not only induce $E\left[e_i(s_i, \pi_i, \theta_i)\right] = s_i$ but it will also induce $e_i(s_i, \pi_i, \theta_i) = s_i$ (perfect compliance with certainty). Therefore, we can write (31) as:

$$\left.\frac{E\left[\frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial s_i}\right]}{E\left[\frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial \pi_i}\right]}\right|_{e_i(s_i,\pi_i,\theta_i)=s_i} = \frac{P(\theta_i < \theta_{J_i}) + P(\theta_i = \theta_{J_i}) \times \frac{\pi_{J_i} f''(0)}{c_i''(s_i,\theta_{J_i})+\pi_{J_i} f''(0)}}{P(\theta_i = \theta_{J_i}) \times \frac{-f'(0)}{c_i''(s_i,\theta_{J_i})+\pi_{J_i} f''(0)}}$$

and (30) as:

$$\frac{E\left[\frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial s_i}\right]}{E\left[\frac{\partial e_i(s_i,\pi_i,\theta_i)}{\partial \pi_i}\right]}_{e_i(s_i,\pi_i,\theta_i)=s_i} =$$

$$\frac{\beta_i \pi_{J_i} f'(0) - \lambda_2^i - Cov\left[c_i'(s_i,\theta_i), \frac{\partial e_i(s_i,\pi_{J_i},\theta_i)}{\partial s_i}\right]}{-\mu_i - Cov\left[c_i'(s_i,\theta_i), \frac{\partial e_i(s_i,\pi_{J_i},\theta_i)}{\partial \pi_i}\right]}$$

Combining these two equations, we obtain:

$$\frac{P(\theta_i < \theta_{J_i}) + P(\theta_i = \theta_{J_i}) \times \frac{\pi_{J_i} f''(0)}{c_i''(s_i,\theta_{J_i}) + \pi_{J_i} f''(0)}}{P(\theta_i = \theta_{J_i}) \times \frac{-f'(0)}{c_i''(s_i,\theta_{J_i}) + \pi_{J_i} f''(0)}}$$

$$= \frac{\beta_i \pi_{J_i} f'(0) - \lambda_2^i - Cov\left[c_i'(s_i,\theta_i), \frac{\partial e_i(s_i,\pi_{J_i},\theta_i)}{\partial s_i}\right]}{-\mu_i - Cov\left[c_i'(s_i,\theta_i), \frac{\partial e_i(s_i,\pi_{J_i},\theta_i)}{\partial \pi_i}\right]}$$

or, after operating and using $\lambda_2^i \geq 0$,

$$\left(\frac{P(\theta_i < \theta_{J_i}) + P(\theta_i = \theta_{J_i}) \times \frac{\pi_{J_i} f''(0)}{c_i''(s_i,\theta_{J_i}) + \pi_i f''(0)}}{P(\theta_i = \theta_{J_i}) \times \frac{f'(0)}{c_i''(s_i,\theta_{J_i}) + \pi_{J_i} f''(0)}}\right) \times \left(\mu_i + Cov\left[c_i'(s_i,\theta_i), \frac{\partial e_i(s_i,\pi_{J_i},\theta_i)}{\partial \pi_i}\right]\right)$$

$$\leq \beta_i \pi_{J_i} f'(0) - Cov\left[c_i'(s_i,\theta_i), \frac{\partial e_i(s_i,\pi_{J_i},\theta_i)}{\partial s_i}\right]$$

Q.E.D.

The sign of $Cov\left(c_i'(s_i,\theta_i), \frac{\partial e_i(s_i,\pi_{J_i},\theta_i)}{\partial \pi_i}\right)$: Without loss of generality, we assume that θ_i may take only two possible values: low (θ_L) and high (θ_H). We also assume that $c_i'(s_i,\theta_i)$ is linear. Then, we can write:

$$Cov\left(c_i'(s_i,\theta_i), \frac{\partial e_i(s_i,\pi_H,\theta_i)}{\partial \pi_i}\right) = E\left(\left(c_i'(s_i,\theta_i) - E\left(c_i'(s_i,\theta_i)\right)\right)\left(\frac{\partial e_i(s_i,\pi_H,\theta_i)}{\partial \pi_i} - E\left(\frac{\partial e_i(s_i,\pi_H,\theta_i)}{\partial \pi_i}\right)\right)\right)$$

$$= \left(c_i'(s_i,\theta_L) - c_i'(s_i,\bar{\theta}_i)\right) \times \left(0 - E\left(\frac{\partial e_i(s_i,\pi_H,\theta_i)}{\partial \pi_i}\right)\right) \times P(\theta_i = \theta_L) +$$

$$\left(c_i'(s_i,\theta_H) - c_i'(s_i,\bar{\theta}_i)\right) \times \left(\frac{-f'(0)}{c_i''(s_i,\theta_H) + \pi_H f''(0)} - E\left(\frac{\partial e_i(s_i,\pi_H,\theta_i)}{\partial \pi_i}\right)\right) \times P(\theta_i = \theta_H) +$$

Assuming $\partial c_i'(s_i,\theta_i)/\partial \theta_i < 0$, as we do, and recalling that $E\left(\frac{\partial e_i(s_i,\pi_H,\theta_i)}{\partial \pi_i}\right) < 0$, the first term of the above expression is positive. With respect to the second one,

$c_i'(s_i,\theta_H) - c_i'(s_i,\bar{\theta}_i) < 0$ and rewriting

$$\frac{-f'(0)}{c_i''(s_i,\theta_H)+\pi_H f''(0)} - E\left(\frac{\partial e_i(s_i,\pi_H,\theta_i)}{\partial \pi_i}\right)$$

as

$$\frac{-f'(0)}{c_i''(s_i,\theta_H)+\pi_H f''(0)} - P(\theta_i = \theta_{J_i}) \times \frac{-f'(0)}{c_i''(s_i,\theta_{J_i})+\pi_{J_i} f''(0)} =$$
$$\frac{-f'(0)}{c_i''(s_i,\theta_H)+\pi_H f''(0)} \times (1 - P(\theta_i = \theta_{J_i}))$$

it is easy to see that this expression is also negative.

Therefore, we can conclude that $Cov\left(c_i'(s_i,\theta_i), \frac{\partial e_i(s_i,\pi_{J_i},\theta_i)}{\partial \pi_i}\right)$ is positive. **Q.E.D.**

The sign of $Cov\left[c_i'(s_i,\theta_i), \frac{\partial e_i(s_i,\pi_{J_i},\theta_i)}{\partial s_i}\right]$: Similarly to what we assume in the above proof, we assume that θ_i may take only two possible values: low (θ_L) and high (θ_H) and that $c_i'(s_i,\theta_i)$ is linear. Then, we can write

$$Cov\left(c_i'(s_i,\theta_i), \frac{\partial e_i(s_i,\pi_H,\theta_i)}{\partial s_i}\right) =$$
$$E\left(\left(c_i'(s_i,\theta_i) - E\left(c_i'(s_i,\theta_i)\right)\right)\left(\frac{\partial e_i(s_i,\pi_H,\theta_i)}{\partial s_i} - E\left(\frac{\partial e_i(s_i,\pi_H,\theta_i)}{\partial s_i}\right)\right)\right)$$

$$\left(c_i'(s_i,\theta_L) - c_i'(s_i,\bar{\theta}_i)\right) \times \left(1 - E\left(\frac{\partial e_i(s_i,\pi_H,\theta_i)}{\partial s_i}\right)\right) \times P(\theta_i = \theta_L) +$$
$$\left(c_i'(s_i,\theta_H) - c_i'(s_i,\bar{\theta}_i)\right) \times \left(\frac{\pi_H f''(0)}{c_i''(s_i,\theta_H)+\pi_H f''(0)} - E\left(\frac{\partial e_i(s_i,\pi_H,\theta_i)}{\partial s_i}\right)\right) \times P(\theta_i = \theta_H) +$$

Assuming $\partial c_i'(s_i,\theta_i)/\partial\theta_i < 0$, as we do throughout, and recalling that $0 < E\left(\frac{\partial e_i(s_i,\pi_H,\theta_i)}{\partial s_i}\right) < 1$, the first term of the above expression is positive. With respect to the second one, $c_i'(s_i,\theta_H) - c_i'(s_i,\bar{\theta}_i) < 0$ and rewriting

$$\frac{\pi_H f''(0)}{c_i''(s_i,\theta_H)+\pi_H f''(0)} - E\left(\frac{\partial e_i(s_i,\pi_H,\theta_i)}{\partial s_i}\right)$$

as

$$\frac{\pi_H f''(0)}{c_i''(s_i,\theta_H)+\pi_H f''(0)} - \left(P(\theta_i = \theta_L) + P(\theta_i = \theta_{J_i}) \times \frac{\pi_H f''(0)}{c_i''(s_i,\theta_H)+\pi_H f''(0)}\right) =$$
$$\frac{\pi_H f''(0)}{c_i''(s_i,\theta_H)+\pi_H f''(0)} \times (1 - P(\theta_i = \theta_{J_i})) - (P(\theta_i = \theta_L)) =$$
$$\left(\frac{\pi_H f''(0)}{c_i''(s_i,\theta_H)+\pi_H f''(0)} - 1\right) \times (P(\theta_i = \theta_L)) < 0$$

Therefore, we can conclude that $Cov\left(c_i'(s_i,\theta_i), \frac{\partial e_i(s_i,\pi_{J_i},\theta_i)}{\partial s_i}\right)$ is positive. **Q.E.D.**

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